Large-$N$ quantum field theories

Lecture course by Dr. P. V. Buividovich,
e-mail: pavel.buividovich@physik.uni-regensburg.de

Abstract

In Quantum Chromodynamics, the theory of strong interactions, perturbation theory fails at low energy due to the large value of the coupling constant. However, as first noticed by t’Hooft, the inverse of the number of quark colours, $1/N_c = 1/3$, can be regarded as an alternative small expansion parameter. Since then the expansion in the number $N$ of field components has become an extremely useful tool in modern theoretical physics, with applications ranging from condensed matter theory to quantum gravity. In many cases, one can find an explicit analytical solution in the limit of infinite $N$. This lecture course reviews some basic ideas underlying such analytical solutions as well as methods for performing practical calculations. Basic knowledge of quantum field theory is required.

Brief outline of lectures

- Saddle point approximation and collective field variables
- Diagram counting for matrix-valued fields and random surfaces
- Analytic solutions of some simple matrix models
- Large-$N$ Schwinger-Dyson equations
- Non-Abelian gauge theories in the large-$N_c$ limit
- Loop equations in non-Abelian large-$N_c$ gauge theories
- Eguchi-Kawai reduction of large-$N_c$ gauge theories

Detailed program with references

1. Saddle point approximation and collective field variables
   - Saddle point approximation on the example of the Stirling formula
2. **Diagram counting for matrix-valued fields and random surfaces**

- Perturbation theory in simple matrix models, diagram counting, double-line notation, genus and topology [2, 3]
- Matrix models as the models of 2D quantum gravity [3]
- Single-trace and multi-trace observables and their interpretation in terms of open random surfaces
- Real and complex matrix ensembles and their interpretation [2, 4, 5]
- Weingarten model of random surfaces on the cubic lattice [6]

3. **Analytic solutions of some simple matrix models**

- Angular coordinates in matrix space (hermitian and unitary), Van der Monde determinant [2]
- Resolvents and eigenvalue density
- “Coulomb gas” interpretation of the Van der Monde determinant
- Explicit evaluation of eigenvalue density for some simple matrix models: Wigner semicircle, $\phi^4$ theory [2, 3]
- Convergence of planar perturbation theory and asymptotic character of $1/N$ expansion
- 3d-order phase transition in Gross-Witten matrix model, weak- and strong-coupling expansions [1, 7]

4. **Large-$N$ Schwinger-Dyson equations**

- Schwinger-Dyson equations in a simple large-$N$ $\phi^4$ theory and their explicit solution
• Schwinger-Dyson equations for simple matrix models and their relation to eigenvalue density [1, 3]
• Schwinger-Dyson equations for Weingarten model [6, 8, 9]
• Interpretation of Schwinger-Dyson equations in terms of random surfaces

5. **Non-Abelian gauge theories in the large-\(N_c\) limit**

• Diagram counting in large-\(N_c\) gauge theories
• t’Hooft and Veneziano limits
• Phenomenology of large-\(N_c\) gauge theories: Zweig rule etc. [10]
• Various approaches to baryons in large-\(N_c\) gauge theories [11]
• Large-\(N\) limit of lattice gauge theories

6. **Loop equations in non-Abelian large-\(N_c\) gauge theories** [1, 12]

• The notion of loop space, Wilson loops as a full set of observables, Mandelstam constraints, “zigzag symmetry”
• Physical observables (field strength, chiral condensate etc.) in terms of loop variables
• Functional derivatives of loop variables, simplifications in the large-\(N\) limit [13]
• Non-Abelian Stokes theorem
• Area derivative and its regularizations
• QCD loop equations, maybe with fermions

7. **Eguchi-Kawai reduction of large-\(N_c\) gauge theories**

• Derivation of loop equations in large-\(N\) lattice gauge theory and their geometric interpretation [1, 14]
• Eguchi-Kawai reduction [14]
• Momentum-space interpretation of EK matrices [12, 15]
• Statistical interpretation of large-\(N\) diammatics [12, 15]
• Quenched and twisted EK models [12, 15–17]
References


