Lattice QCD at finite temperature

V. G. Bornyakov

CFS, Protvino & Moscow

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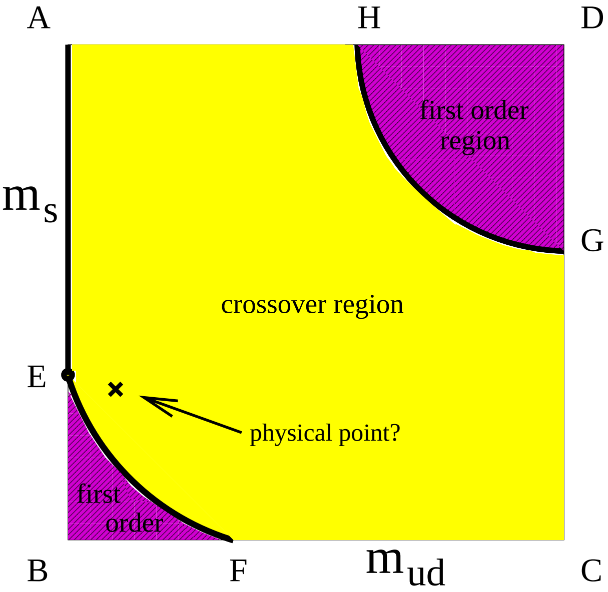
Outline

1 – Critical temperature

2 – Screening masses and spatial string tension

3 – Conclusions and perspectives

Critical temperature



DESY - ITEP - Kanazawa collaboration

DESY, Zeuthen:

Y. Nakamura, G. Schierholz, V. Weinberg

ITEP, Moscow & IHEP, Protvino:

VB, S. Morozov, M. Polikarpov

Kanazawa University:

T. Suzuki, Y. Koma

- $N_f = 2$ lattice QCD

- Lattice action:
  - Wilson gauge field action
  - Improved Wilson fermionic action

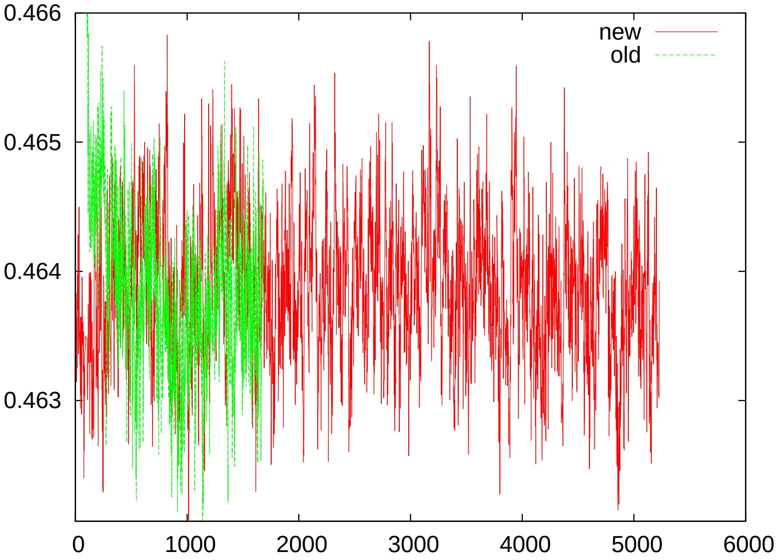
$$S_F = S_F^{(0)} - \frac{\mathrm{i}}{2}\kappa\, g\, c_{sw} a^5 \sum_s \bar{\psi}(s)\sigma_{\mu\nu}F_{\mu\nu}(s)\psi(s)$$

- Lattice size $N_t \times N_s^3$:
  - $N_t = 8, \quad N_s = 16$
  - $N_t = 10 \quad N_s = 24$
  - $N_t = 12 \quad N_s = 24$

- Physical parameters:
  - 500 MeV $\lesssim$ m_π $\lesssim$ 1100 MeV
  - 2000 MeV $\lesssim$ $1/a$ $\lesssim$ 2500 MeV
- m_π and $1/a$ were obtained by interpolation/extrapolation of $T = 0$ results of QCDSF-UKQCD collaboration
- Polyakov loop susceptibility χ_L is used to determine psudocritical point

$\beta = 5.2$, $24^3\,10$				$\beta = 5.29$, $24^3\,12$			
κ	Traj.	disc.	τ	κ	Traj.	disc.	τ
0.1348	940	260	0.5	0.1357	2860	500	1.0
0.1352	2960	1300	0.5	0.1358	3910	1000	1.0
0.1353	3795	1000	0.5	0.1359	4800	2000	1.0
0.1354	3645	1500	1.0	0.1360	4355	1000	1.0
0.1355	10000	2300	1.0	0.1361	1540	2000	1.0
0.1356	10000	3000	1.0				
0.1357	9240	1500	1.0				
0.1358	6500	2500	1.0				
0.1359	3280	500	1.0				
0.1360	2800	1500	1.0				

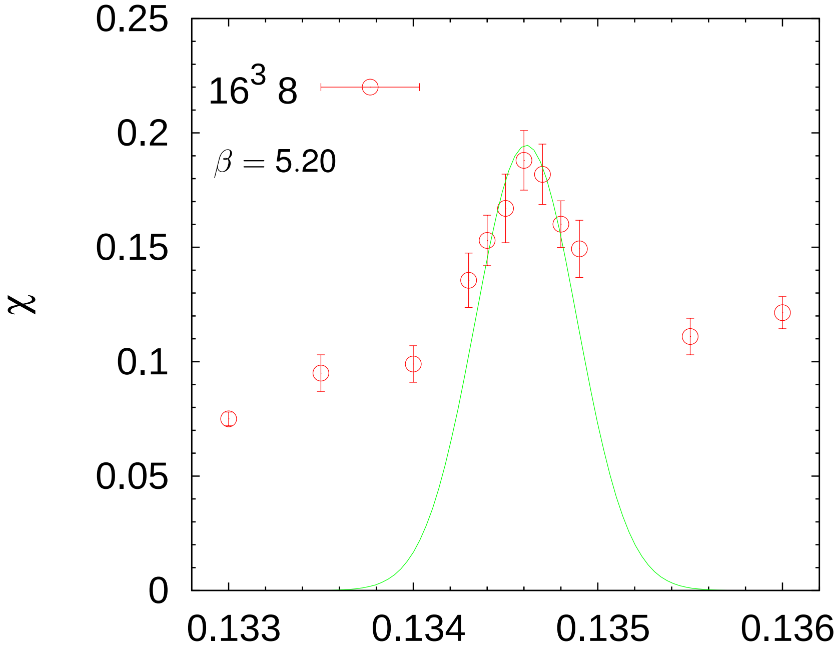
Parameters and statistics of the simulation including the length of the trajectory τ for $N_t = 10$ and $N_t = 12$ lattices.

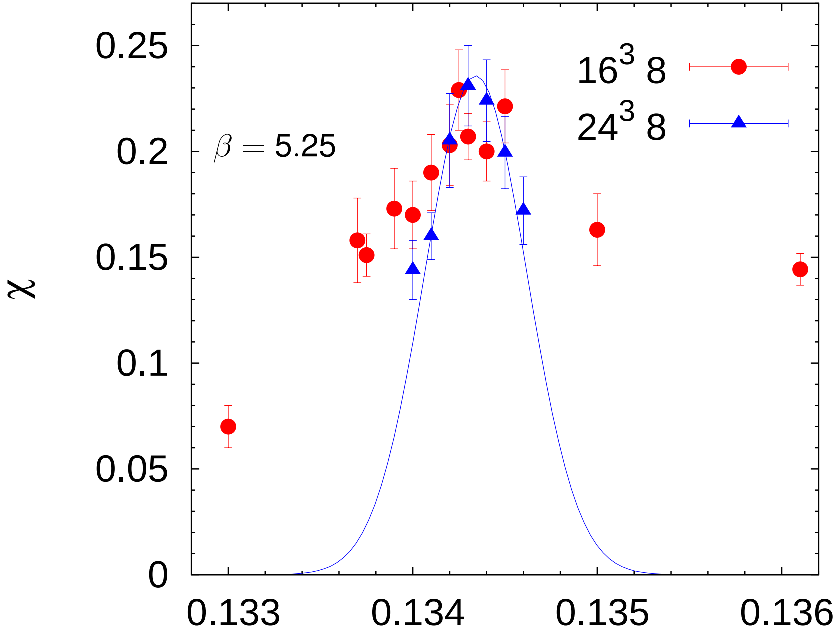


Plaquette history, $\beta = 5.25, \kappa = 0.1354$, lattice $24^3 \times 10$.

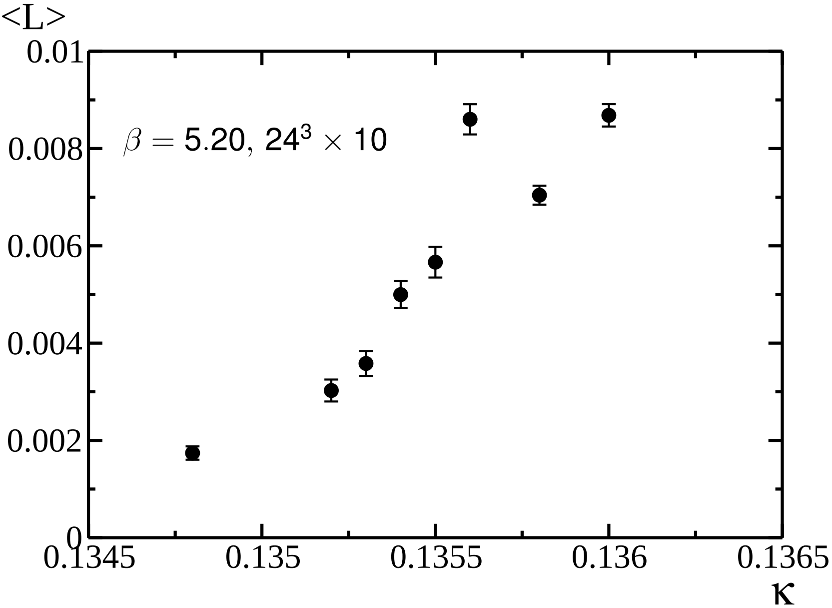
$\beta = 5.2$, $16^3\,8$			$\beta = 5.25$, $16^3\,8$			$\beta = 5.25$, $24^3\,8$		
κ	Traj.	τ	κ	Traj.	τ	κ	Traj.	τ
0.1330	8780	0.25	0.1330	1800	0.25	0.1340	3014	1.0
0.1335	6000	0.25	0.1337	3515	1.0	0.1341	4180	1.0
0.1340	5500	0.25	0.13375	9920	0.25	0.1342	9240	1.0
0.1343	3560	1.0	0.1339	9400	0.25	0.1343	9273	1.0
0.1344	4600	1.0	0.1340	10600	0.25	0.1344	10134	1.0
0.1345	9200	1.0	0.1341	8000	0.25	0.1345	11995	1.0
0.1346	9600	1.0	0.1342	13600	0.25	0.1346	5140	1.0
0.1347	8224	1.0	0.13425	15023	1.0			
0.1348	7270	1.0	0.1343	15000	1.0			
0.1349	4260	1.0	0.1344	9615	1.0			
0.1355	6350	0.25	0.1345	16017	1.0			
0.1360	4163	1.0	0.1350	1680	1.0			
			0.1361	4208	1.0			

Parameters and statistics of the simulation including the length of the trajectory τ for $N_t = 8$ lattices.

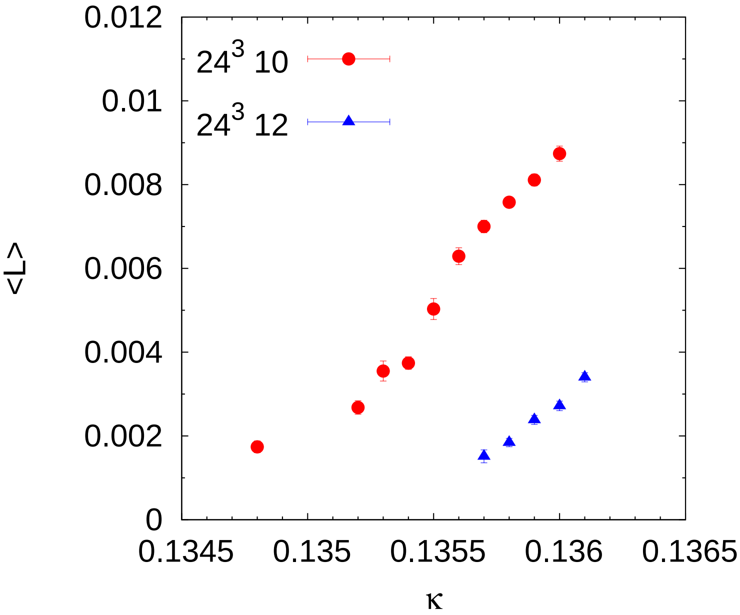

16^3 8
β = 5.20
χ
0.25
0.2
0.15
0.1
0.05
0
0.133
0.134
0.135
0.136

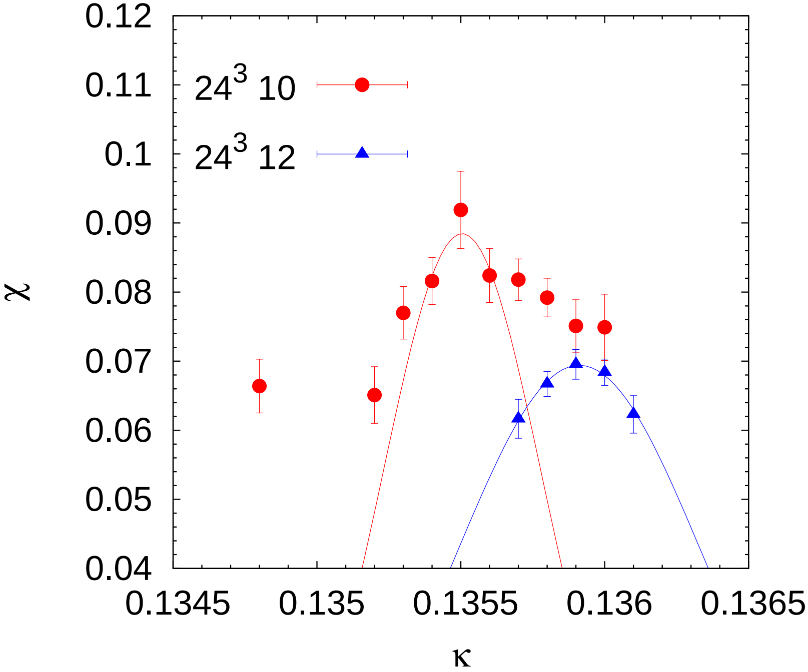


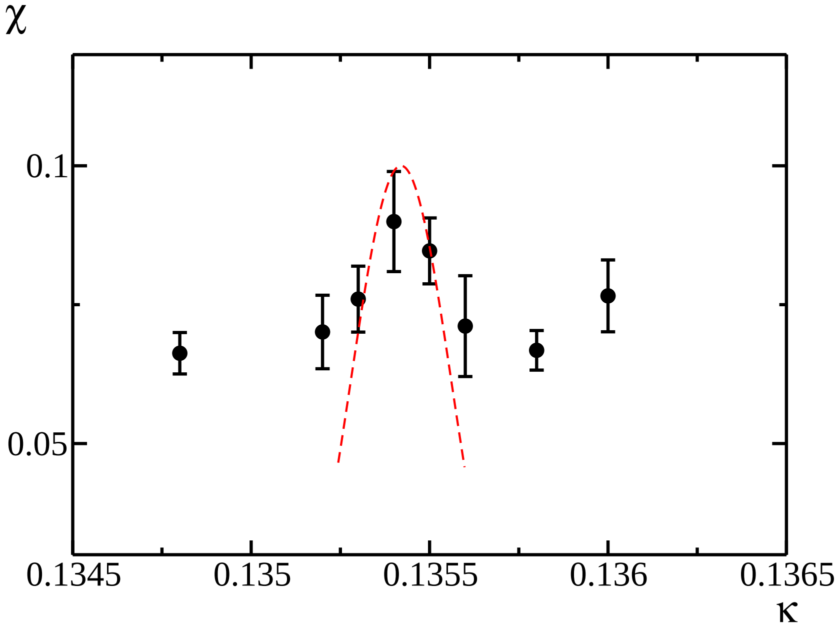
0.25
0.2
0.15
0.1
0.05
0
χ
β = 5.25
16³ 8
24³ 8
0.133
0.134
0.135
0.136



<L>
0.01
0.008
0.006
0.004
0.002
0
$\beta = 5.20,\ 24^3 \times 10$
0.1345
0.135
0.1355
0.136
0.1365
κ


0.012
0.01
0.008
0.006
0.004
0.002
0
0.1345
0.135
0.1355
0.136
0.1365
κ
<L>
24^3 10
24^3 12


0.12
0.11
0.1
0.09
0.08
0.07
0.06
0.05
0.04
χ
24^3 10
24^3 12
0.1345
0.135
0.1355
0.136
0.1365
κ



Lattice $24^3\,10$

Fitting function

$$r_0 T_c(r_0 m_\pi, 1/N_t) = r_0 T_c(0,0) + c_N \cdot \frac{1}{N_t^2} + c_m \cdot (r_0 m_\pi)^d \qquad (1)$$

d=1.08 for universality class of 3d O(4) symmetric spin model.

Another possibility for continuum limit extrapolation

$$r_0 T_c(r_0 m_\pi, a/r_0) = r_0 T_c(0,0) + c_a \cdot \left(\frac{a}{r_0}\right)^2 + c_m \cdot (r_0 m_\pi)^d \qquad (2)$$

Difference between results of fits (1) and (2) gives estimation of respective systematic error.

Old result of fit (1):

$$r_0 T_c(r_0 m_\pi^{ph}, 0) = 0.438(6)(-7)(+13) \quad (3)$$

In physical units

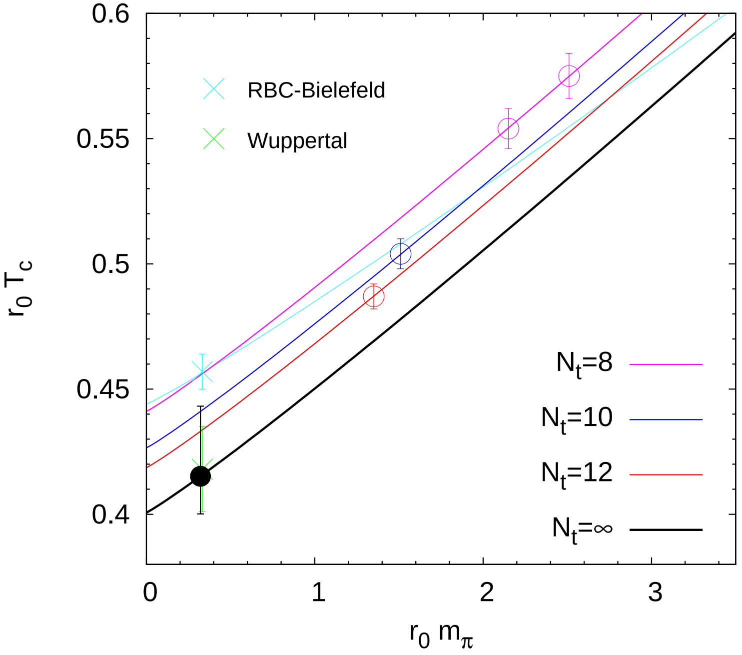
$$\begin{aligned} T_c &= 173(2)(-2)(+4) MeV, \; r_0 = 0.5 fm \\ &= 185(2)(-2)(+4) MeV, \; r_0 = 0.467 fm \end{aligned} \quad (4)$$

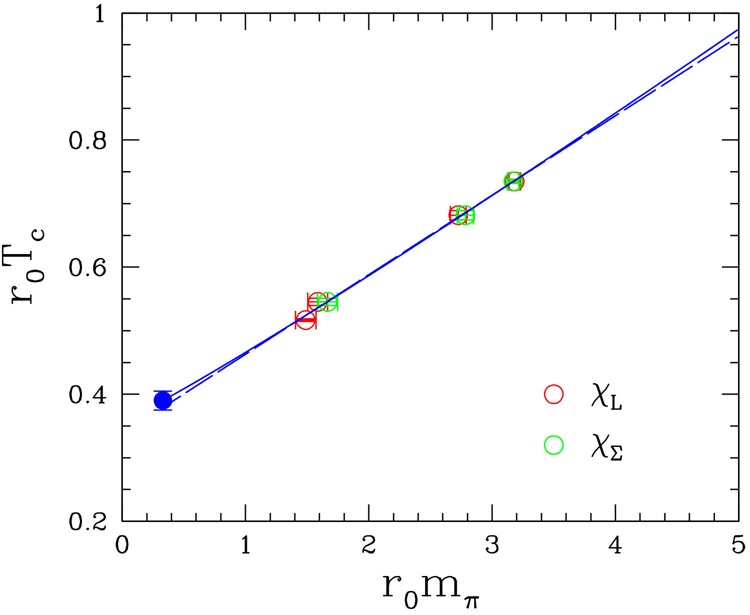
New result of fit (1):

$$r_0 T_c(r_0 m_\pi^{ph}, 0) = 0.415(3)(-15)(+38) \quad (5)$$

$$T_c = 164(1)(-5)(+10) MeV, \; r_0 = 0.5 fm \quad (6)$$

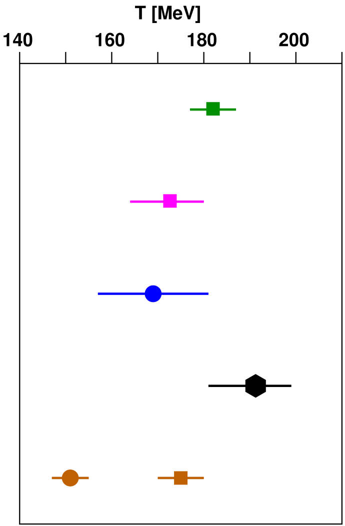
$$= 176(1)(-5)(+10) MeV, \; r_0 = 0.467 fm \quad (7)$$


0.6
0.55
0.5
0.45
0.4
0
1
2
3
$r_0 T_c$
$r_0 m_\pi$
RBC-Bielefeld
Wuppertal
N_t=8
N_t=10
N_t=12
N_t=∞



1
0.8
0.6
0.4
0.2
0
1
2
3
4
5
$r_0 T_c$
$r_0 m_\pi$
χ_L
χ_Σ

Where are we now?



use T=0 scale: r0=0.469fm

Nf=2:

V.G. Bornyakov et al, POS Lat2005, 157 (2006)
(improved Wilson, Nt=8, 10; input: r0=0.5 fm)
(added Nt=12, Lattice'07) (rescaled to r0)

Y. Maezawa et al., hep–lat/0702005 (QM'2006)
(improved Wilson, Nt=4, 6; input: m–rho)
(no cont. exp. yet)

Nf=2=1:

C. Bernard et al., Phys.Rev. D71, 034504 (2005)
(improved staggered (asqtad), Nt=4,6,8, input r1)
(rescaled to r0)

M. Cheng et al., Phys.Rev D74, 054507 (2006)
(improved staggered (p4), Nt=4,6; input r0)

Y. Aoki et al., Phys. Lett. B643, 46 (2006)
(staggered (stout), Nt=4,6,8,10; input fK)
(converted to r0)

chiral deconfinement chiral+deconfinement

Screening masses at $T > T_c$

Free energy in different color chanels:

$$
\begin{aligned}
e^{-F_1(R,T)/T} &= \frac{1}{3}\langle\ \mathrm{Tr} L^\dagger(x) L(y)\ \rangle \\
e^{-F_8(R,T)/T} &= \frac{1}{8}\langle\ \mathrm{Tr} L^\dagger(x)\ \mathrm{Tr} L(y)\ \rangle - \frac{1}{24}\langle\ \mathrm{Tr} L^\dagger(x) L(y)\ \rangle \\
e^{-F_6(R,T)/T} &= \frac{1}{12}\langle\ \mathrm{Tr} L(x)\ \mathrm{Tr} L(y)\ \rangle + \frac{1}{12}\langle\ \mathrm{Tr} L(x) L(y)\ \rangle \\
e^{-F_3^*(R,T)/T} &= \frac{1}{6}\langle\ \mathrm{Tr} L(x)\ \mathrm{Tr} L(y)\ \rangle - \frac{1}{6}\langle\ \mathrm{Tr} L(x) L(y)\ \rangle
\end{aligned}
$$

Nadkarni (1986)

Fit at large RT:

$$V_M(R,T) \equiv F_M(R,T) - F_M(\infty,T) = -C_M \frac{\alpha(T)}{R} e^{-m_D(T)R}$$

- Coulomb gauge — Philipsen (2002)
  Iterative overrelaxation gauge fixing procedure with one gauge copy;
  check of Gribov copies effects with 3 random gauge copies

- Hypercubic blocking — Hasenfratz and Knechtly (2001)
  HCB decreases statistical errrors by factor 3

Previous studies in $N_f = 2$ QCD

Kaczmarek and Zantow (2005)

staggered fermions, $N_t = 4; \quad m_\pi / m_\rho = 0.7$

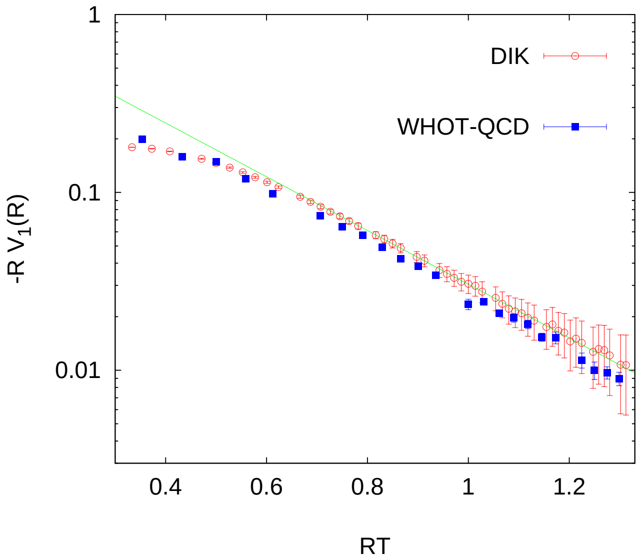
$$\frac{m_D}{T} = A \left(1 + \frac{N_f}{6}\right)^{1/2} g_{two-loop}(T) , \qquad A \approx 1.4$$

WHOT-QCD (2007)

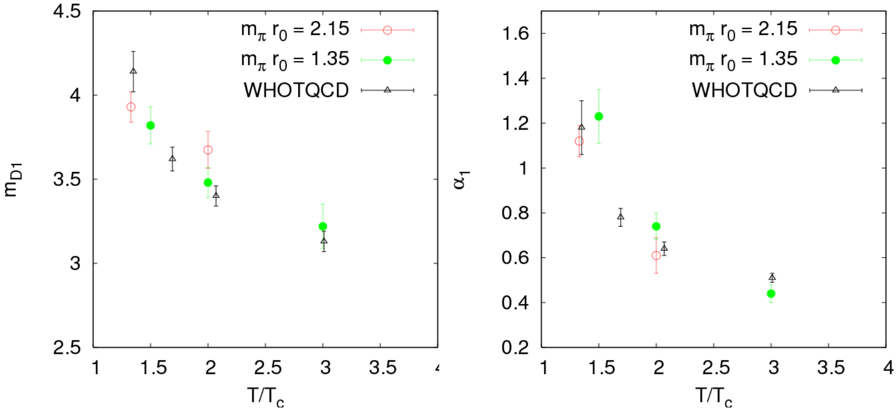
- improved Wilson fermions, $N_t = 4$; $m_\pi/m_\rho = 0.65, 0.80$
- Casimir scaling for $V_M(R,T)$
- phenomenological relation:

$$\frac{m_D}{T} = \left(1 + \frac{N_f}{6}\right)^{1/2} \sqrt{4\pi\alpha(T)}$$

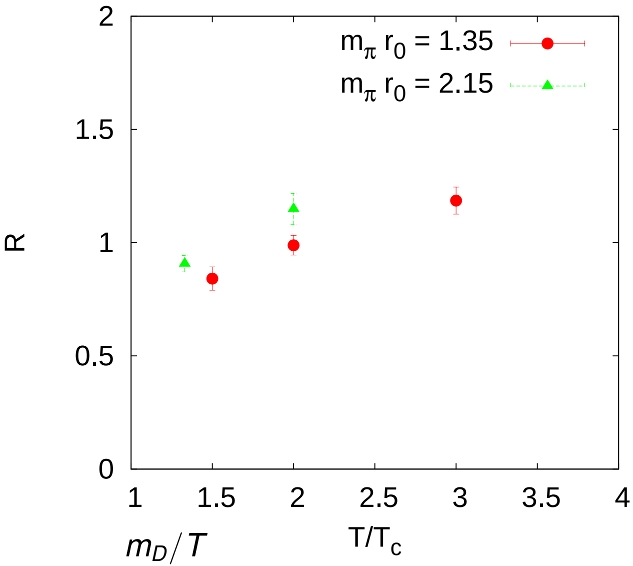
- Comparison with K&Z : agreement for $\alpha(T)$, 20% deviation for $m_D(T)$
- Too coarse lattices ?



Comparison with Tsukuba for $T/T_c = 2$



The singlet mass m_{D1}/T and singlet coupling α_1 compared with Tsukuba.



$$R = \frac{m_D/T}{\left(1+\frac{N_f}{6}\right)^{1/2}\sqrt{4\pi\alpha(T)}}$$

Spatial string tension

Spatial static potential $V_s(R)$

$$aV_s(R) = \lim_{Z\to\infty} \log \frac{W(R,Z)}{W(R,Z+1)}\,, \tag{8}$$

$W(R,Z)$ are Wilson loops of size $\frac{R}{a} \times \frac{Z}{a}$

Ansatz:

$$V_s(R) = V_0 - \alpha/R + \sigma_s R\,. \tag{9}$$

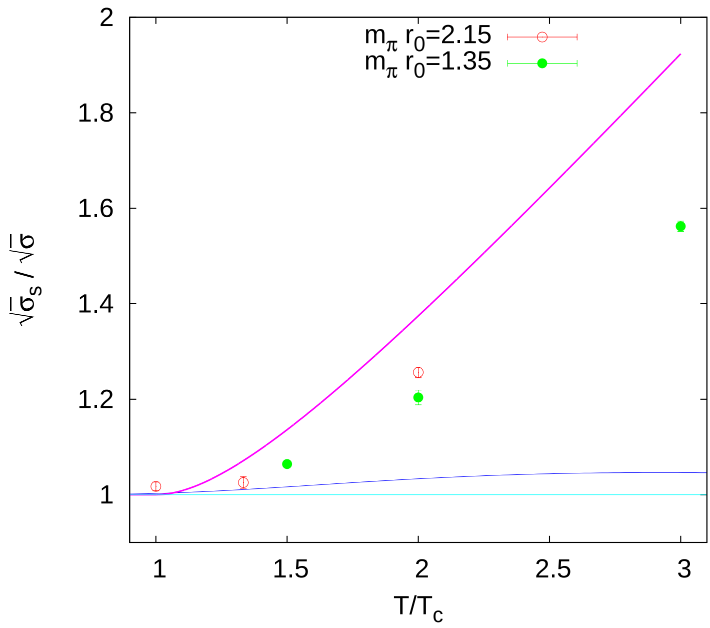
temperature interval: $1 \; < \; T/T_c \; < \; 3$

Theoretical predictions:
1)Field correlators approach, N. O. Agasian, 2003

$$\frac{\sigma_s(T)}{\sigma} = \frac{\sinh(M/T) + M/T}{\cosh(M/T) + 1}, \qquad (10)$$

for $M/T_c >> 1$.
Here $M = 1.5$ Gev (M/T_c=7.5)
2)AdS/QCD approach, O. Andreev and V. I. Zakharov, 2007

$$\frac{\sigma_s(T)}{\sigma} = \left(\frac{T}{T_c}\right)^2 \exp\left[\left(\frac{T_c}{T}\right)^2 - 1\right], \quad T \geq T_c, \qquad (11)$$



2
1.8
1.6
1.4
1.2
1
$\sqrt{\sigma_s} / \sqrt{\sigma}$
$m_\pi r_0$=2.15
$m_\pi r_0$=1.35
1
1.5
2
2.5
3
T/T_c

Conclusions and perspectives

- T_c in the continuum limit at physical m_π is closer to Wuppertal result;
  warning message for RBC-Bielefeld concerning their continuum extrapolation
- Screening masses at T/T_c up to 3 are in good agreement with WHOT-QCD results confirming disagreement with staggered fermions results
- Spatial string tension for T near to T_c is practically constant and approximately equal to $T = 0$ string tension
  Agreement with theoretical prediction for $T \gtrsim T_c$

- Computations at smaller quark masses are on the way on lattices $24^3 \times 12$ and $32^3 \times 12$. The goal is to produce two more data points at $m_\pi r_0$ about 1.0 and 0.8 thus improving our etrapolation to the physical mass.
- Simulations of finite temperature $N_f = 2 + 1$ QCD with improved Wilson fermions and improved Symanzik gauge field action are to be started this year.